Statistics

Lecture 20

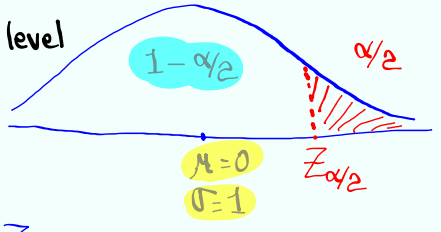
Feb 19-8:47 AM

$Z_{\alpha/2}$ is a critical value such that the area to its right is $\alpha/2$.

α Significance level

α Alpha

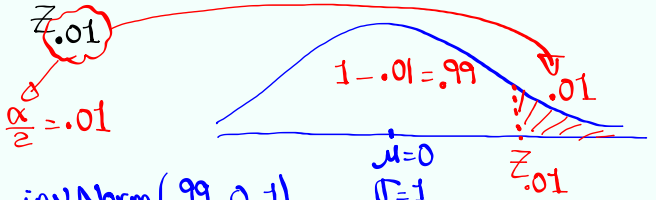
$0 < \alpha < 1$



How to find $Z_{\alpha/2}$

use invNorm(left Area, μ, σ)

Find $Z_{.01}$

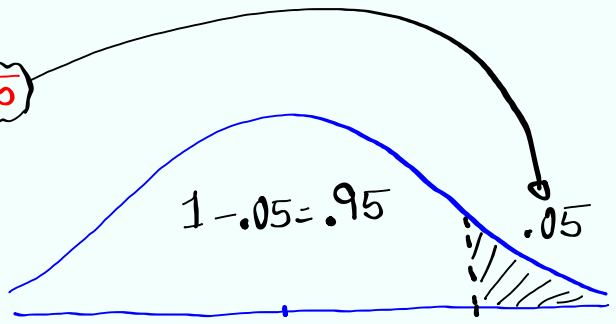


$Z_{.01}$ = invNorm(.99, 0, 1) ≈ 2.326

Apr 30-1:55 PM

Find $z_{\alpha/2}$ for $\alpha = .1$.

$\alpha/2 = .05 \rightarrow z_{.05}$



$z_{.05} = \text{invNorm}(.95, 0, 1) \approx \boxed{1.645}$ $\mu = 0$, $\sigma = 1$ $\quad z_{.05}$

If α not given $\Rightarrow$ use .05

Apr 30-2:01 PM

α Significance level

$0 < \alpha < 1$

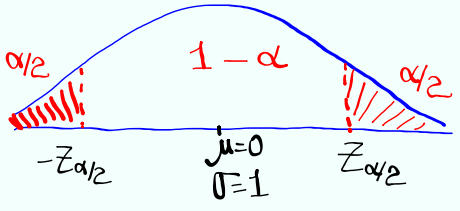
$\alpha/2$ is the area of each tail

$(1 - \alpha) \cdot 100\%$ Confidence level

$\alpha = .02$

$1 - \alpha = .98 = 98\%$ Confidence level

Think of Conf. level as the middle area under the curve



Apr 30-2:04 PM

Find $\pm z_{\alpha/2}$ for 98% C-level

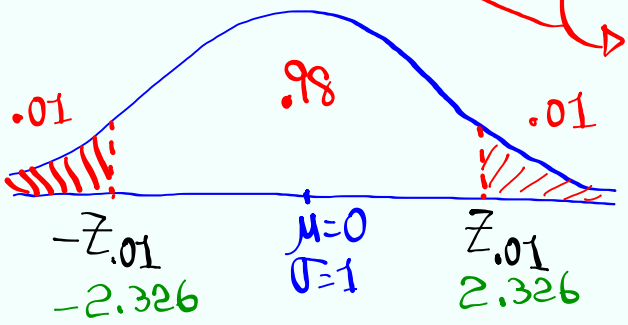
Confidence level

Middle area is .98

$1-\alpha = .98$

$\alpha = .02$

$\alpha/2 = .01$



$z_{.01} = \text{invNorm}(.99, 0, 1) = 2.326$

Apr 30-2:08 PM

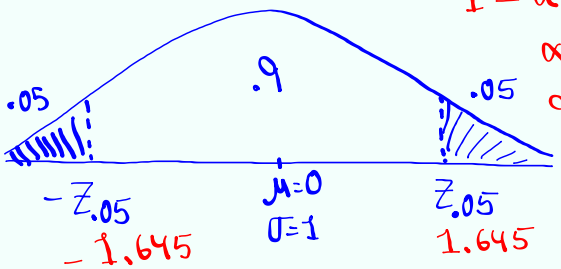
Find $\pm z_{\alpha/2}$ for 90% C-level.

Middle Area .9

$1-\alpha = .9$

$\alpha = .1$

$\alpha/2 = .05$



$z_{.05} = \text{invNorm}(.95, 0, 1) \approx 1.645$

If α not given $\Rightarrow$ use .05

If C-level not given $\Rightarrow$ use 95%

Apr 30-2:12 PM

Estimating Parameters:

Parameters describe Population while Statistic ↓ Sample.

we use statistic to guess the Corresponding Parameter.

we use $\hat{P}$ (P-hat) Sample Proportion to estimate Population Proportion P.

we use $\bar{x}$ Sample Mean to estimate Population Mean μ.

we use S Sample Standard deviation to estimate Population Standard deviation σ.

To estimate	we use	
P	$\hat{P}$	Point-Estimate
μ	$\bar{x}$	
σ	S	

Apr 30-2:17 PM

when estimating parameters, the answer is a range of values.

Confidence interval

Probability that the parameter falls within Confidence interval is the Confidence level.

Middle Area

Apr 30-2:25 PM

Suppose the Conf. interval for population mean is 72 to 92 with 95% C-level.

$P(72 < \mu < 92) = .95$

Suppose the Conf. interval for population Proportion is .38 to .46 with 90% C-level

$P(.38 < p < .46) = .9$

C-level is the probability that the Parameter falls in the Confidence interval.

Given $P(12 < \sigma < 20) = .98$

Prob. that pop. standard deviation falls between 12 and 20 is .98

Apr 30-2:28 PM

Estimating Population Proportion

$$\hat{P} - E < p < \hat{P} + E$$

- p: Population Proportion
- $\hat{P}$: Sample Proportion, Point-estimate
- E: Margin of error

$$\hat{p} = \frac{x}{n}$$

- x: # of favorable responses
- n: Sample Size

$$\hat{q} = 1 - \hat{p}$$

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

$z_{\alpha/2}$ is the Critical Value for $(1-\alpha)\cdot 100\%$ C-level.

Apr 30-2:35 PM

I randomly selected 100 voters and 80 of them were in support of gun control.

$n = 100$ $\quad \hat{p} = \frac{x}{n} = \frac{80}{100} = .8$

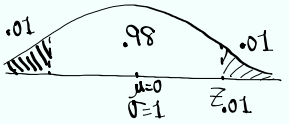
$x = 80$ $\quad \hat{q} = 1 - \hat{p} = .2$ **C-level: .98**

I want to find **98% Conf. interval** for the prop. of all voters in support of gun control.

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

$= 2.326 \cdot \sqrt{\frac{(.8)(.2)}{n}}$

$\approx .09$



$z_{.01} = \text{invNorm}(.99, 0, 1) = 2.326$

$\hat{p} - E < p < \hat{p} + E$

$.8 - .09 < p < .8 + .09$

$.71 < p < .89$

we are 98% confident that between 71% and 89% of all voters are in support of gun control.

Apr 30-2:41 PM

Using TI:

STAT → TESTS ↓ 1-PropZInt

$x = 80$

$n = 100$

C-level: .98

Calculate

$.70695 < p < .89305$

$.71 < p < .89$

$\hat{p} = \frac{.89 + .71}{2} = .8$

$E = \frac{.89 - .71}{2} = .09$

Apr 30-2:50 PM

I surveyed 250 students and 32 of them were smokers.
$n = 250$
$x = 32$

Find 99% Conf. interval for the prop. of all students that smoke.

C-level: .99

1-PropZInt
$x: 32$
$n = 250$
C-level: .99
Calculate

$.074 < p < .182$

$.07 < p < .18$

I am 99% confident that between 7% and 18% of all students smoke.

$\hat{p} = \frac{.18 + .07}{2} = .125$
Point-estimate

$E = \frac{.18 - .07}{2} = .055$
Margin of error

Apr 30-2:55 PM

I surveyed 425 LA residents and 60% of them say Lakers are going to win tonight.
$n = 425$
$\hat{p} = .6$
$\hat{p} = \frac{x}{n}$
$x = n\hat{p}$
$x = 425(.6) = 255$
if decimal → Round-up

Find Confidence interval for the prop. of all LA residents that have same feeling.

$.55 < p < .65$

NO C-level
⇒ use .95

1-PropZInt
$x = 255$
$n = 425$
C-level: .95
Calculate

$\hat{p} = \frac{.65 + .55}{2} = .6$

$E = \frac{.65 - .55}{2} = .05$

Apr 30-3:03 PM

How to determine the Sample Size needed:

$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$

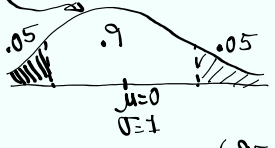
with some algebra $n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$

Always round-up to a whole #.

Suppose 80% of students have iphone.

Find min. Sample Size needed to Construct 90% Conf. interval for the prop. of all students that have iPhone and error not to exceed 4%.

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$



$= (.8)(.2)\left(\frac{1.645}{.04}\right)^2$

$z_{.05} = \text{invNorm}(.95,0,1) = 1.645$

$= 270.6025$

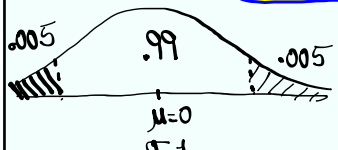
$n = 271$

Apr 30-3:12 PM

Find minimum Sample Size needed to Construct 99% Conf. interval for the Prop. of all users of a new app. Called ChatNow and error not exceed 5%.

$\hat{p}$ & $\hat{q}$ are both unknown use .5 for each

$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$



$= (.5)(.5)\left(\frac{2.576}{.05}\right)^2$

$1 - .99 = .01$

$= 663.5776$

$.01 \div 2 = .005$

$n = 664$

$z_{.005} = \text{inv Norm}(.995,0,1) = 2.576$

Apr 30-3:21 PM

When working with Conf. interval for Proportion

1) use invNorm to find $Z_{\alpha/2}$

2) use 1-PropZInt to find Conf. interval.

3) Looking for $x \Rightarrow x = n\hat{p}$ Always round-up

4) C-level not given $\Rightarrow$ use .95

5) $\hat{p} = \frac{+}{2}$, $E = \frac{-}{2}$

6) Min. Sample Size $n = \hat{p}\hat{q}\left(\frac{Z_{\alpha/2}}{E}\right)^2$

$\hat{p}$ & $\hat{q}$ not given $\Rightarrow$ use .5 for each — Always Round-up

Apr 30-3:30 PM

I surveyed 324 students, 22% of them were fan of online classes.

$n = 324$, $\hat{p} = .22$ $\rightarrow x = n\hat{p} = 324(.22) = 71.28 \rightarrow x = 72$

Find 98% conf. interval for the prop. of all students that are fan of online classes.

$.17 < p < .28$

C-level: .98

1-PropZInt

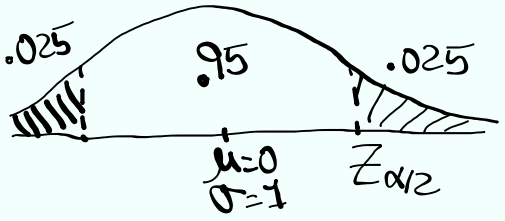
$\hat{p} = \frac{.28 + .17}{2} = .225$

$E = \frac{.28 - .17}{2} = .055$

Apr 30-3:34 PM

How many should we survey if we wish to be 95% conf. and error to be within 4%?

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$



$z_{.025} = \text{invNorm}(.975, 0, 1) \approx \boxed{1.960}$

$$= (.22)(.78)\left(\frac{1.960}{.04}\right)^2$$

$$= 412.0116$$

$$\boxed{n = 413}$$

Apr 30-3:40 PM